Heat Engine : Carnot Heat Engine ① TDC I. Paper II

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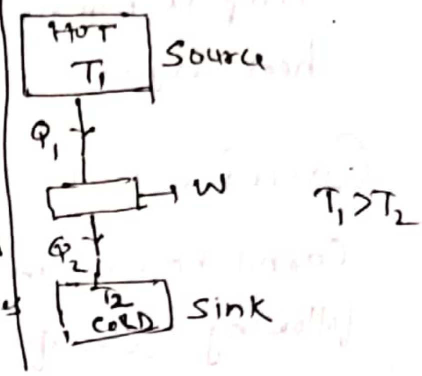
Heat engines:- Heat engines are those machines which are used to convert heat energy into mechanical work. During operation, a heat engine absorbs heat at higher temperature, converts a part of heat absorbed into mechanical work and rejects remaining heat at lower temperature. In this process, a working substance is used. In steam engine, the working substance is water vapour and in all gas engines, the working substance is a combustible mixture of gases.

In any heat engine, the working substance goes through certain changes of pressure, volume and temperature and then it returns back to the initial state. The complete changes through which the working substance undergoes from its initial state and back to its starting state, ~~Ex~~ constitutes one complete cycle of operation.

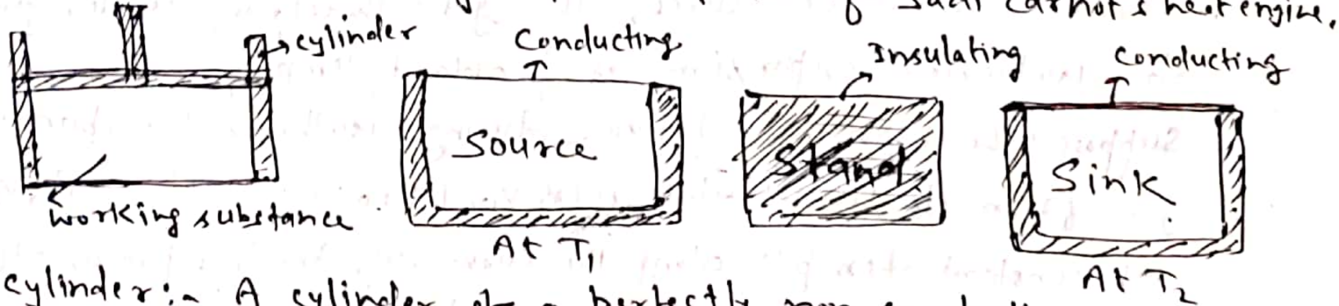


Definition of Efficiency of Heat Engine:- The ratio of the mechanical work done by the engine in one cycle to the heat absorbed by engine from the high temperature source, is known as efficiency of the heat engine. It is denoted by η.

$$\text{Efficiency } \eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = \frac{\text{Mechanical work done}}{\text{Heat absorbed}}$$

Since $Q_1 - Q_2 < Q_1$ so efficiency can never be 100%.

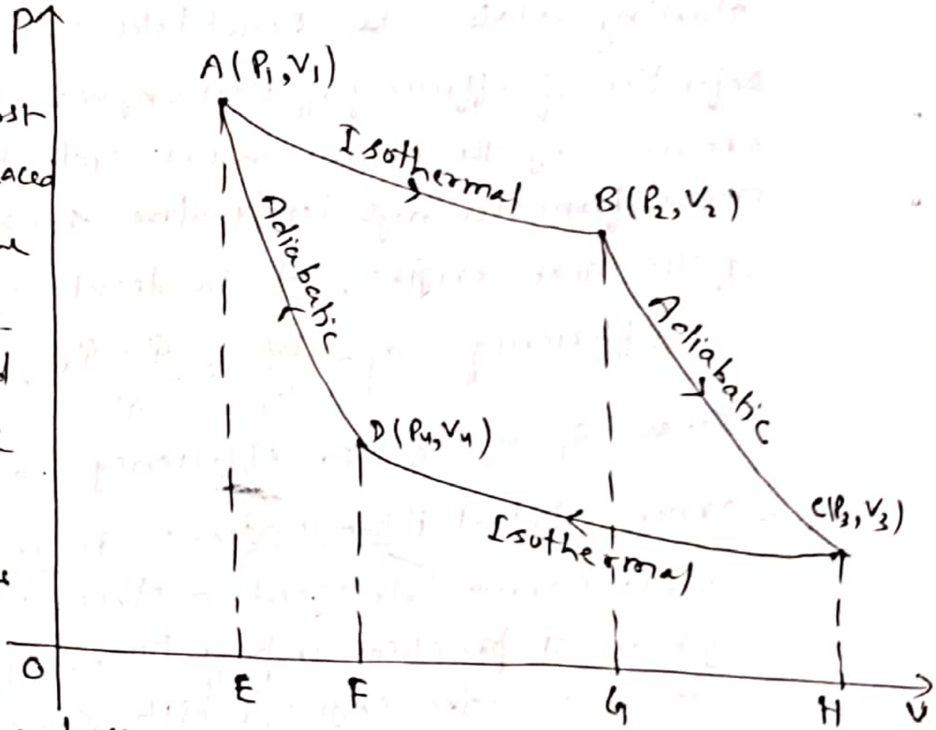
Carnot's Ideal Heat Engine:- In 1824, the French Engineer Sadi Carnot designed a theoretical engine which is free from all practical imperfections. It has maximum efficiency. It is an ideal engine which can not be realised in practice. There are following important parts of Sadi Carnot's heat engine.



1. **Cylinder:-** A cylinder of a perfectly non conducting wall, a perfectly conducting base with a non conducting piston (which moves without friction in the cylinder) contains one mole of perfect gas as the working substance.

2. Source: A reservior (source) of infinite thermal capacity whose temperature is maintained at constant temp T_1 from which the engine can draw any amount of heat at constant temperature T_1 by perfect conduction.
3. Heat insulating stand:- A perfectly non conducting platform which works as an insulating stand for adiabatic process.
4. Sink: Sink is a reservoir of infinite thermal capacity whose temp is maintained at constant lower temp T_2 ($T_2 < T_1$) to which heat engine can reject any amount of heat.

Carnot's cycle: for obtaining continuous supply of work from Carnot heat engine, the working substance has to pass through the following cycles of quasi-static operation known as Carnot's cycle shown in figure.



1. Isothermal expansion: First of all, the cylinder is placed on the source so that the gas acquires the temp T_1 then after it is allowed to undergo quasi-static expansion. As the gas expands, its temp tends to fall but heat passes into the cylinder through the perfectly conducting base in contact with the source. The gas therefore, undergoes slow isothermal expansion at constant temp T_1

Suppose the working substance during isothermal expansion goes from its initial state A (P_1, V_1, T_1) to the state B (P_2, V_2, T_1) at constant temp T_1 along the curve AB. In this process, the working substant absorbs heat Q_1 from the source at constant temp T_1 and it does work W_1 which is given by

$$Q_1 = W_1 = \int_{V_1}^{V_2} P\,dV = RT_1 \log_e\left(\frac{V_2}{V_1}\right) = \text{Area of ABGEA.}$$

2. Adiabatic expansion:- Now the cylinder is removed from the source and it is placed on the insulating stand. The working substance gas is allowed to go slow adiabatic expansion from the state $B(P_2, V_2, T_1)$ to the state $C(P_3, V_3, T_2)$ along ~~BC~~ the curve BC performing external work at the expense of its internal energy untill its temp falls to T_2 temp of the sink. There is no transfer of heat in the process BC. Work done W_2 in the adiabatic expansion process is given by

$$W_2 = \int_{V_2}^{V_3} P\,dV = K\int_{V_2}^{V_3} \frac{1}{V^\gamma}\,dV \qquad \because PV^\gamma = K \Rightarrow P = \frac{K}{V^\gamma}$$

$$= K\left[\frac{V^{-\gamma+1}}{-\gamma+1}\right]_{V_2}^{V_3} = \frac{K}{1-\gamma}\left[\frac{V}{V^\gamma}\right]_{V_2}^{V_3} = \frac{1}{1-\gamma}\left[\frac{K}{V_3^\gamma}\cdot V_3 - \frac{K}{V_2^\gamma}\cdot V_2\right]$$

$$= \frac{1}{1-\gamma}\left[P_3V_3 - P_2V_2\right] \qquad \because P_2 = \frac{K}{V_2^\gamma},\ P_3 = \frac{K}{V_3^\gamma}$$

$$W_2 = \frac{RT_2 - RT_1}{1-\gamma} \qquad \because P_1V_1 = RT_1,\ P_2V_2 = RT_2$$

$$\Rightarrow W_2 = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{Area of BCHGB.}$$

3. Isothermal compression:- Now the cylinder is removed from the insulating stand and it is placed on the sink of temp T_2. Now the piston is slowly moved inwards so that the work is done on the gas. The temp tends to increase due to heat produced by the compression. Since the conducting base of the cylinder is in contact with the sink so the produced heat passes to the sink and temp of the gas remains constant T_2. Thus the gas undergoes isothermal compression starting from the state $C(P_3, V_3, T_2)$ to the state $D(P_4, V_4, T_2)$ at constant temp T_2 along the curve CD and rejects some heat Q_2 to the sink given by

$$Q_2 = W_3 = \int_{V_3}^{V_4} P\,dV = RT_2 \log_e\left(\frac{V_4}{V_3}\right)$$

or $Q_2 = W_3 = -RT_2 \log_e\left(\frac{V_3}{V_4}\right) = -$ Area of CHFDC

where -ve sign indicates that work is done on the working substance.

4. Adiabatic compression:- Now the cylinder is removed from the sink and it is placed on the insulating stand. Now the piston is slowly moved inwards so that the gas is adiabatically compressed and temp of the gas rises. The adiabatic process is continued till the gas comes back to its original state $A(P_1, V_1, T_1)$ for completing one full cycle. This operation is represented by adiabatic curve DA starting from the ~~pou~~ state $D(P_4, V_4, T_2)$ to the final state $A(P_1, V_1, T_1)$. In this process work W_4 is done on the gas is given by

$$W_4 = \int_{V_4}^{V_1} P\,dV = -\frac{R(T_1 - T_2)}{\gamma - 1} = -\text{Area of DFEAD}$$

where -ve sign indicates that work is done on the gas.

Since W_2 and W_4 are equal and opposite so they cancel each other.

Work done by the engine per cycle is

$$W = W_1 + W_2 + W_3 + W_4$$

$$\Rightarrow W = W_1 + W_3 \qquad \because W_2 = -W_4$$

From the graph, net work done per cycle is

W = Area ABGEA + Area BCHGB − Area CHFDA − Area DFEAD

= Area ABCDA

Thus the area enclosed by the Carnot's cycle consisting of two isothermals and two adiabatics gives net amount of work done per cycle.

In the cyclic process, Net heat absorbed = Net work done per cycle

$$Q_1 - Q_2 = W_1 + W_3 = RT_1 \log_e\left(\frac{V_2}{V_1}\right) - RT_2 \log_e\left(\frac{V_3}{V_4}\right) \quad \text{---} (1)$$

For adiabatic process BC, $T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1}$ ——— (2)

For adiabatic process DA, $T_1 V_1^{\gamma-1} = T_2 V_4^{\gamma-1} \Rightarrow \frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$ ——— (3)

From eqns (2) and (3), $\left(\frac{V_2}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \Rightarrow \frac{V_2}{V_3} = \frac{V_1}{V_4} \Rightarrow \frac{V_3}{V_4} = \frac{V_2}{V_1}$ put in (1)

Net work done $W = Q_1 - Q_2 = RT_1 \log_e\left(\frac{V_2}{V_1}\right) - RT_2 \log_e\left(\frac{V_2}{V_1}\right) = R(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right)$.

Net work done per cycle by the working substance in carnot heat engine is

$$W = Q_1 - Q_2 = R(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right) \quad \text{———— (4)}$$

Heat absorbed by the working substance from the source at higher temp. T_1 is

$$Q_1 = RT_1 \log_e\left(\frac{V_2}{V_1}\right) \quad \text{———— (5)}$$

Efficiency of the carnot heat engine:- The ratio of mechanical work done by the working substance in one complete cycle to the heat absorbed by the working substance from the source at higher temp. T_1, is known as efficiency of the carnot heat engine.

Thus efficiency $\eta = \frac{W}{Q_1} = \frac{\text{work done by working substance}}{\text{Heat absorbed by working substance}}$

$$\Rightarrow \eta = \frac{Q_1 - Q_2}{Q_1} = \frac{R(T_1 - T_2)\log_e\left(\frac{V_2}{V_1}\right)}{RT_1 \cdot \log_e\left(\frac{V_2}{V_1}\right)}$$

$$\Rightarrow \eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

or $$\boxed{\eta = \left(1 - \frac{T_2}{T_1}\right) = \left(1 - \frac{Q_2}{Q_1}\right)} \quad \text{———— (6)}$$

where Q_2 = heat rejected by the working substance to the sink at lower temp T_2.

From the formula (6) of efficiency for carnot heat engine, it is clear that efficiency of the carnot heat engine depends only upon temperatures of source (T_1) and sink (T_2). It does not depend upon nature of the working substance. Efficiency of the engine is always less than unity because $T_1 > T_2$.

If the temp. of the sink $T_2 = 0$ then $\eta = 100\%$

But it is not possible to attain absolute zero temp (0K) so 100% conversion of heat into mechanical work by carnot heat engine is not possible.

* If $T_1 = T_2$ then efficiency $\eta = 0$ i.e., if temperature of source be equal to temperature of the sink then efficiency of the engine will become zero. That means the carnot heat engine does not work.

Effective way to increase Efficiency of carnot heat engine

The efficiency of carnot heat engine is

$$\eta = 1 - \frac{T_2}{T_1}$$

From formula of efficiency, it is clear that for increasing efficiency η, of carnot's engine,

(1) Temperature T_2 of the sink is kept as low as possible by keeping temp T_1 of source, constant.

or (2) Temperature T_1 of the source is kept as high as possible by keeping constant temp T_2 of the sink.

In practice, it is not convenient to use the heat sink at a temp. below the temp. of the surrounding (atmosphere). Therefore, the more effective way to increase the efficiency of carnot's engine, is to use the heat source at a temperature as high as possible.

Carnot's heat enegine & Refrigerator

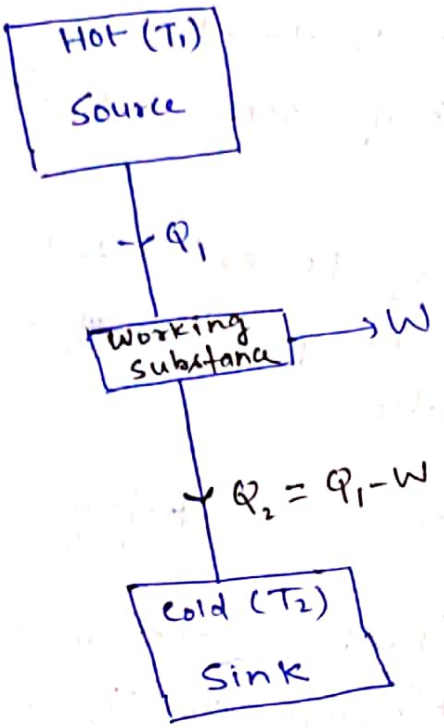


Fig-1: Heat engine

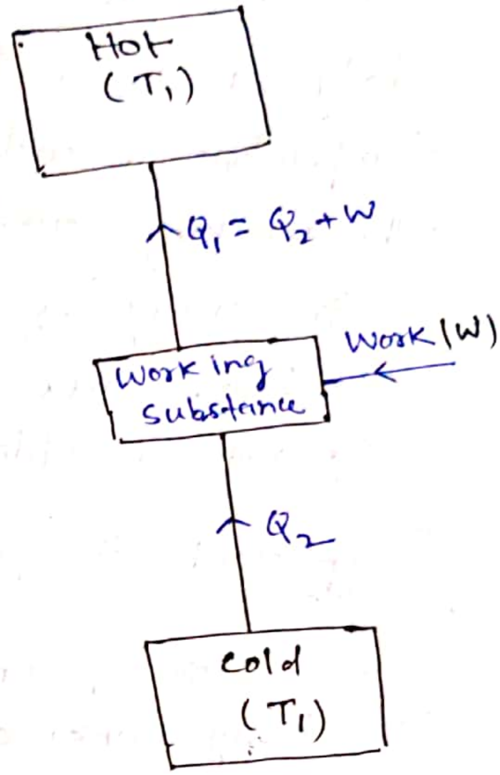


Fig-2: Refrigerator

Carnot's cycle is perfectly reversible process. It can work as heat engine and it can also work as refrigerator,

* When it is operated as heat engine then it absorbs heat Q_1 from the source at higher temp T_1. mechanical work W is done by the working substance and it rejects heat $Q_2 = Q_1 - W$ to the sink at lower temp T_2 $(T_1 > T_2)$ as shown in fig-1.
* When it is operated as a refrigerator then it absorbs heat Q_2 from the sink at lower temp T_2, mechanical work W is done on the working substance by some external agency and it reject heat $Q_1 = Q_2 + W$ to the source at higher

temp T_1 $(T_1 > T_2)$ as shown in fig. 2.

* In the case of refrigerator, heat flows from the body at lower temperature T_2 to the body at higher temperature T_1 with the help of some external work on the working substance.

In every cycle, heat Q_2 is extracted from the cold body. This will not be possible if the cycle is not completely reversible.

Coefficient of performance of a Refrigerator

The ratio of amount of heat Q_2 absorbed from the cold body (sink) at constant temperature T_2 to the amount of work done (W) on the working substance by an external agency, is known as coefficient of performance of the refrigerator.

Coefficient of performance of a refrigerator is

$$\boxed{P = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}}$$

where Q_2 = Amount of heat absorbed from cold body at constant lower temp. T_2

Q_1 = Amount of heat rejected to the hot body at constant higher temp T_1

$W = Q_1 - Q_2$ = Work done on the working substance by external agency.

If the working substance be an ideal gas then

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

Subtracting 1 from both sides

$$\frac{Q_1}{Q_2} - 1 = \frac{T_1}{T_2} - 1 \Rightarrow \frac{Q_1 - Q_2}{Q_2} = \frac{T_1 - T_2}{T_2}$$

or $$\frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}$$

Thus, coefficient of performance of the refrigerator will be

$$\boxed{P = \frac{Q_2}{Q_1 - Q_2} = \frac{T_2}{T_1 - T_2}}$$

or $$\boxed{P = \frac{1}{\frac{Q_1}{Q_2} - 1} = \frac{1}{\frac{T_1}{T_2} - 1}}$$

Second Law of Thermodynamics

There are following statements of Second law of thermodynamics given by different Scientists.

Lord Kelvin's statement:- "It is impossible to get a continuous supply of work from a body by cooling it to a temperature lower than the temperature of its surrounding".

Kelvin-planck's statement:- "It is impossible to construct an engine which is operating in cycle, will take heat from a body (source) and convert it completely into work without leaving any change any where"

Clausius's Statement :- "It is impossible for a self acting machine working in a cyclic process unaided by external agency to transfer heat from a body at lower temperature to the body at higher temperature".

or " It is impossible to construct a device which operating in a cycle, will transfer heat from a cold body to a hot body without expenditure of work by some external agency i.e, heat can never flow spontaneously from a cold body to a hot body".